Fundamentals of Electric Theory and Circuits

by

Sridhar Chitta

IK International Publishing House Pvt. Ltd.
4435-36/7, Ansari Road, Daryaganj,
New Delhi – 110002, INDIA
Email: info@ikinternational.com   Website: www.ikbooks.com
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Waveguide Attenuation and Impedance

Number references in square brackets [ ] *are listed in "*Fundamentals of Electric Theory and Circuits". Where not explicitly indicated, Chapter and Section references are in the textbook *"*Fundamentals of Electric Theory and Circuits"

In the following descriptions the reader should pay careful attention to the differences in the transmission characteristics of parallel *planes* and parallel *plate* guides. A parallel conducting plane has an infinitely large extent while a parallel plate has finite dimensions. The plane and the plate may have infinite conductivity or finite conductivity.
Jordan and Balmain (Ref.[16]) in the discussions of wave propagation in waveguides highlight the effects that these guides with walls of infinite conductivity (perfectly conducting or lossless) *planes* or *plates* have on the transmission characteristics. The reader should be aware of these distinctions when understanding the derivations of the field equations in their textbook.

The following discussion is aimed to provide a visualization of the mechanism of *attenuation* of electromagnetic waves by waveguides at frequencies below cut-off (f_c) and to intuitively understand wave impedance. The reader is urged to refer to the textbook Ref. [16] for details of the equations of the electric and magnetic fields in waveguides.

Impedance in TE mode and TM mode modeled as high-pass π-section and T-section filters

Wave impedances are defined at a point for a waveguide (Section 7.08 "Wave Impedances" in Ref.[16]).
There is a marked resemblance between the properties of TE and TM waves between parallel planes and the wave impedances and the characteristic impedances of the prototype T or π sections in ordinary filter theory.
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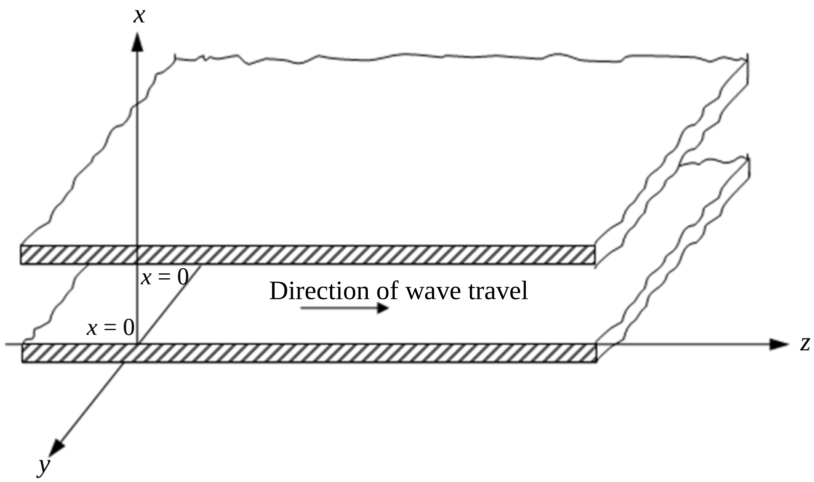
TE mode wave

The wave impedance of a parallel plane (in the *y* and *z* directions) guide with separation between planes '*a*' and propagating a signal of wavelength λ in the *z*-direction in the TE mode is

$$Z_{yx}^{+} = \frac{E_y}{H_x} = \frac{\eta}{\cos\theta} = \frac{\eta}{\sqrt{1-sin^2\theta}} = \frac{\eta}{\sqrt{1-(f_c/f\}^2}} \qquad (1)$$

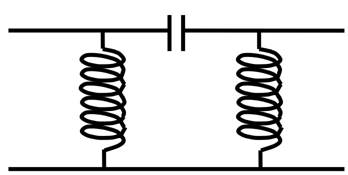
where, $sin\theta = \frac{m\lambda}{2a}$ in which *m* is the mode, $f_c = \frac{1}{\lambda_c\sqrt{\mu\varepsilon}}$, and $\lambda_c = \frac{2a}{m}$.



Parallel conducting planes

The TE wave impedance is real and decreases from an infinitely large value at cut-off toward the asymptotic value of $\eta = \sqrt{\frac{\mu}{\varepsilon}}$, which is the free-space impedance as the frequency increases to values much higher than cut-off (Section 7.08 "Wave Impedances" in Ref.[16]).

The characteristic impedance $Z_{o\pi}$ of a π-Section high-pass filter



π-Section high-pass filter circuit

is given by

$$Z_{o\pi} = \frac{R_o}{\sqrt{1-(f_c/f\}^2}} \qquad (2)$$

where, $R_o = \sqrt{\frac{L}{C}}$, $\omega_c = \frac{1}{2\sqrt{LC}}$, and $f_c = \frac{1}{4\pi\sqrt{LC}}$.

Equation 2 shows that the impedance of the π-Section high-pass filter starts at infinity (∞), is resistive atf_c, and decreases with frequency to approach R_o asymptotically. $Z_{o\pi}$ is resistive in the range $f > f_c$, and reactive for $f < f_c$.

The similarity in form of Eqs (1) and (2) shows that the wave impedance with increasing frequency above cut-off of a wave propagating in the TE mode is similar to that of a π-Section high-pass filter.

For frequencies below cut-off ($f < f_c$), the parallel plane waveguide impedance is a pure reactance and indicates that there is no acceptance of power by the guide and therefore no transmission down the guide.

TM mode wave

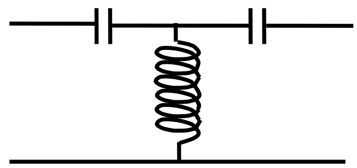
The wave impedance of a parallel plane (in the *y* and *z* directions) guide with separation between planes '*a*' and propagating a signal of wavelength λ in the *z*-direction in the TM mode is

$$Z_{xy}^{+} = \frac{E_x}{H_y} = \eta\, cos\theta = \eta\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad (3)$$

where, $sin\theta = \frac{m\lambda}{2a}$ in which m is the mode, $f_c = \frac{1}{\lambda_c\sqrt{\mu\varepsilon}}$, and $\lambda_c = \frac{2a}{m}$.

The wave impedance varies from zero at the cut-off frequency up to the asymptotic value η for frequencies much higher than cut-off. For frequencies below cut-off ($f < f_c$), the parallel plane waveguide impedance is a pure reactance and indicates that there is no acceptance of power by the guide and therefore no transmission down the guide.

This corresponds to the expression for characteristic impedance of the prototype T-section of a high-pass filter.



T-Section high-pass filter circuit

The characteristic impedance Z_{oT} of a T-Section high-pass filter is given by

$$Z_{oT} = R_o\sqrt{1 - \left(\frac{f_c}{f}\right)^2} \qquad (4)$$

where, $R_o = \sqrt{\frac{L}{C}}$, $\omega_c = \frac{1}{2\sqrt{LC}}$, and $f_c = \frac{1}{4\pi\sqrt{LC}}$.

Equation 4 shows that the impedance of the T-Section high-pass filter is resistive and increasing for $f > f_c$, and zero at $f = f_c$. The characteristic impedance is reactive for $f < f_c$. The characteristic impedance starts at zero at f_c and increases with frequency f to approach R_o asymptotically.

The similarity in form of Eqs (3) and (4) shows that the wave impedance with increasing frequency above cut-off of a wave propagating in the TM mode is similar to that of a T-Section high-pass filter.

Guided wave field equations and power loss in a plane conductor

Several times when discussing the flow of power in waveguides and the power loss in conductor wall surfaces, it is seen that the electric and magnetic fields are expressed in simple phasor notations and the modes are expressed as "when the spacing between plates is an integral number of half-wavelengths". The reader should note that this method which assumes the plates to be perfect conductors (or lossless) enables the field equations to be derived easily and fairly

accurately. If the derivations were to be attempted assuming lossy conductor walls, the equations would become unwieldy and complex.
When the power loss calculations are being made however, engineers impose the conditions of a conductor having a surface resistance R_s (*lossy*).
Authors may not state explicitly that the reasons for obtaining the field equations assuming *lossless* conductors are to simplify the derivation. See "Power loss in a simple resonator" in the Section 6.04 "Power Loss in a plane conductor" in Ref.[16]).

Attenuation when $f < f_c$, penetration depth and skin depth (Refer Section 5.06 "Conductors and Dielectrics" in Ref.[16])

It would be instructive to review the effects of static electric fields in conductors and insulators in Chapter 1, Section 1.14 and how charges spread when placed on an insulated conductor in Chapter 2, Section 2.21 before reading the following.

In a medium which has conductivity the wave is attenuated as it progresses owing to the losses which occur. In a good conductor at radio frequencies the rate of attenuation is very great and the wave may penetrate only a very short distance before being reduced to a negligibly small percentage of its original strength. A term that has significance under such circumstances is the depth of penetration.
Assume that the electromagnetic wave is incident on a solid block of metal. Then the depth of penetration is defined as that depth in which the wave has been attenuated to 1/*e* or approximately 37 percent of its original value. Since the amplitude decreases by the factor $e^{-\alpha x}$ it is apparent that at that distance x, which makes $\alpha x = 1$, the amplitude is only 1/*e* times its value at $x = 0$. By definition this distance is equal to δ, the depth of penetration; so

$$\alpha\,\delta = 1 \text{ or } \delta = \frac{1}{\alpha}.$$

For a good conductor the depth of penetration

$$\delta = \frac{1}{\alpha} \cong \sqrt{\frac{2}{\omega\mu\sigma}}$$

where μ is the permeability of the conductor and σ the conductivity. The penetration depth is commonly called the *skin depth* δ. The derivation of the expression for δ is given in Section 5.06 "Conductors and Dielectrics" in Ref.[16]. Also see Figure 7-11 in Section 7.09 "Electric Field and Current Flow within the Conductor" in Ref. [16] of the instantaneous current distribution within a copper conductor as a 100-MHz wave is guided over its surface.

Example 1

As an example of the order of magnitude of δ in metals, the depth of penetration of a 1MHz wave into copper which has a conductivity 5.8×10^{7} mhos per meter and a permeability approximately equal to that of free space is 0.0667 mm and at 100 MHz is 0.00667 mm.

One consequence of finite conductivity is that any surface current penetrates metals to the skin depth. If the conductor is assumed to be perfect then σ is infinitely large which means that δ the depth of penetration is small. In this case the surface currents will be confined to a thin layer on the surface walls of the guide.

Practical waveguides are typically constructed using walls of metals (good conductors) with air as the medium in which the waves travel. Since the waves between the plates are confined by the walls on which the waves are incident or reflected it may be expected that the fields will penetrate the walls with their depth of penetration characterized by the factor δ.
Above the cut-off frequency f_c, Jordan and Balmain Ref.[16] characterize the wave attenuation by the factor α (i.e for the propagating wave at frequencies above f_c), and below the cut-off frequency by the factor $\bar{\alpha}$.

It will be instructive to review the discussions in Chapter 2 Section 2.9 how the net fields in the wires are reduced by accumulated charges on capacitor plates and in Chapter 5 Section 5.16 by the induced emf in inductors (solenoids) before reading the following.

We discuss attenuation and power loss below the cut-off frequency in two simple cases i) parallel-plane assuming perfectly conducting (lossless) planes and ii) parallel-plane with finite conductivity. Refer Section 7.06 "Velocities of propagation" in Ref. [16].

i) With perfectly conducting planes, below the cut-off frequency, the waves bounce back and forth between the walls of the guide and there is no wave motion parallel to the axis. At the cut-off frequency the phase velocity is zero and propagation of energy along the guide ceases. This is not to say that there are no fields within the guide. The fields *penetrate into the guide* [along the direction of propagation] *with an exponential decrease in amplitude in the z-direction*, and with no phase shift (for the infinitely long guide with perfectly conducting walls). See Section 7.07 "Attenuation in parallel-plane guides" in Ref. [16].
For the TE wave, the impedance is purely reactive at frequencies below cut-off and the phase shift characterized by $\bar{\beta}$ in the wave equation $E_x = E_o sin(\omega t - \bar{\beta} z)$ for a wave travelling in the *z*-direction, is zero. The guide does not accept power for transmission down the guide ($\bar{\beta}$ = 0).
The currents are in a thin sheet on the surface of the plane and the energy remains within a very small distance without propagating. If the walls are perfectly conducting (lossless) planes, decay of the steady-state wave occurs but there is no dissipation of energy. The transmitted power is zero which implies that the attenuation constant $\bar{\alpha}$ will be extremely large.
We had discussed in "How lossless lines also produce attenuation or amplification" towards the end of Section 5.22 Chapter 5 how lumped pure reactances attenuate signals by *pure reactances* and not by dissipative losses in resistances (assuming zero resistance). The development of fringe fields and induced fields which oppose signal voltages in (ideal or lossless) capacitors and (ideal or lossless) inductors is what enabled this process of attenuation. In quite the same manner in the guide with perfectly conducting planes operating below the cut-off frequency, the electromagnetic wave is attenuated without dissipative losses in what is called the *evanescent* mode.
A wave that decays in a direction for which the power flow is purely reactive is designated an *evanescent* wave: (from MIT, USA notes)
1) the field distribution at the origin simply decays exponentially with distance *z* and the fields lose their wave character since they wax and wane in synchrony at all positions, 2) the electric and magnetic fields vary 90 degrees out of phase so that the total energy storage alternates twice per cycle between being purely electric and purely magnetic, and 3) the energy flux becomes purely reactive since the real (time average) power flow is zero.

Above the cut-off frequency, wave propagation does occur and the attenuation of the wave is zero for perfectly conducting planes (See Section 7.04 “Characteristics of TE and TM waves” in Ref. [16]).

ii) If the planes have a finite conductivity, then for frequencies below cut-off there will be attenuation as discussed and dissipative losses as well. The formula for power lost for conducting *plates* is given by

$W_L = 2A\varepsilon|E_z|^2\sqrt{\frac{2\omega}{\mu\sigma}}$ where A is the area of the plates, ε is the permittivity of free space, μ is the permeability of the medium and σ the conductivity of the medium. For copper at 1 MHz, $\sigma \cong 5.8\times10^{7}$, $\mu = \mu_0$ and $\varepsilon = \varepsilon_0$.

We can obtain the power loss per unit area (Watts/square meter) by computing $\frac{W_L}{A}$.

Above the cut off frequency, the wave travels and such a wave that decays with distance from an interface and propagates power parallel to it is called a *surface* wave.

When a block of conductor is immersed in an electric field, the net field inside the conductor is zero. Then why does the penetration depth of the fields reduce as the frequency is increased ?

In the static case of immersing a conductor in a field, the mobile electrons have enough time to arrange themselves on the surface to produce a zero resultant electric field inside. But, *the component fields*, the field in which the conductor is immersed and the field due to the electrons after they arrange themselves *are present* everywhere inside the conductor. The inductive effects are ignored in the steady-state.

When a time-varying field say, a sinusoidally varying field is applied however, there is an inductive effect which should be accounted for. The effect is large when the frequency is high and therefore, the depth of penetration of the field itself becomes smaller and as the frequency is increased the depth becomes still smaller as was seen in the Example 1.

In the sinusoidal steady-state the incident time-varying field is attenuated sharply even as it attempts to penetrate the conductor.

The π-Section high-pass filter circuit shows an inductor at the input port suggesting that the input impedance will be zero or negligibly small for slowly varying signals. On the other hand for frequencies below the cut-off frequency the impedance is merely shown as a region of no wave propagation in Fig. 7-8 of wave impedances in Ref. [16]. Why is this so ?

The model of a π-Section high-pass filter circuit equivalent of a parallel-plane waveguide operating in the TE mode is not accurate in all aspects and is applicable only for frequencies above cut-off. Therefore, an interpretation of the wave impedance from a purely equivalent circuit viewpoint will be counter-intuitive.

Similarly, the model of the T-Section high-pass filter circuit equivalent of a parallel-plane waveguide operating in the TM mode is not accurate in all aspects and is applicable only for frequencies above cut-off and interpreting the wave impedance from a purely equivalent circuit viewpoint will be counter-intuitive.